Why you should treat your ML defense like a *theorem*

Florian Tramèr Stanford \rightarrow Google \rightarrow Vacation \rightarrow ETHZ

Defense⇔**Theorem**

Theorem 7.7. $P \neq NP$.



Defense⇔Theorem, Evaluation⇔Proof.

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$$\alpha_1\beta\gamma_1$$
 $\alpha_2\beta\gamma_2$ $\alpha_1\beta\gamma_2$ $\alpha_2\beta\gamma_1.$

Note that since O(n) variables have to be changed when jumping from one cluster to another, we may even chose our $poly(\log n)$ blocks to be in overlaps of these variables. This would mean that with a $poly(\log n)$ change in frozen variables of one cluster, we would get a solution in another cluster. But we know that in the highly constrained phases of d1RSB, we need O(n) variable flips to get from one cluster to the next. This gives us the contradiction that we seek.



6 ADAPTIVE ATTACK EVALUATION

Given that our proposed defense effectively prevents existing blackbox attacks, we now study whether or not it can prevent more sophisticated attacks. We find that while it is possible to degrade the effectiveness of the defense, we can not defeat it completely. We study three categories of attacks: gradient attacks (specifically, variants of NES with various kinds of query blinding), boundaryfollowing attacks (specifically, variants of the boundary attack with query blinding), and hybrid attacks (which combine gradient attacks with a surrogate mode).

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<u>Today:</u> refuting a defense = building an attack.





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Adversarial Examples Are Not Easily Detected: Bypassing Ten Detection Methods

Nicholas Carlini David Wagner University of California, Berkeley

On Adaptive Attacks to Adversarial Example Defenses

Florian Tramèr^{*} Stanford University

Nicholas Carlini^{*} Wieland Brendel^{*} Google University of Tübin Aleksander Mądry

MIT

SoK: How Robust is Image Classification Deep Neural Network Watermarking? (Extended Version)

Nils Lukas, Edward Jiang, Xinda Li, Florian Kerschbaum

Definition of Security: Circumventing Defenses to Adversarial Examples

Anish Athalye^{*1} Nicholas Carlini^{*2} David Wagner²

Reliable Evaluation of Adversarial Robustness with an Ensemble of Diverse Parameter-free Attacks

Francesco Croce¹ Matthias Hein¹

What's next? Breaking 100 defenses?



This is not how we refute theorems!

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Here's an efficient algorithm for 3-SAT!

Instead, we just refute the proof.

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There's a flaw in line 637... **REJECT!**

Similarly, we should focus on refuting ML defense *evaluations*.



This evaluation is unconvincing because... Conclusion: NOT ROBUST



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What makes an evaluation convincing?





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What makes a *proof* convincing?



Ten Signs a Claimed Mathematical Breakthrough is Wrong



Ten Signs a Claimed Mathematical Breakthrough is Wrong

- 1. The authors don't use TeX.
- 2. The authors don't understand the question.
- 3. The approach seems to yield something much stronger and maybe even false.
- 4. The approach conflicts with a known impossibility result.
- 5. The authors themselves switch to weasel words by the end.
- 6. The paper jumps into technicalities without presenting a new idea.
- 7. The paper doesn't build on (or in some cases even refer to) any previous work.
- 8. The paper wastes lots of space on standard material.
- 9. The paper waxes poetic about "practical consequences".
- 10. The techniques just seem too wimpy for the problem at hand.

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Some of these don't really apply to ML...

```
Theorem 1.1.1.
For every algorithm \omega, which solves the OWMF(F) problem,
the computational complexity of \omega is at least floor (floor(2^{(q-1)/3} / (2^{2/3} + 2^{1/3} + 1)) / 2).
Proof:
Ad absurdum, suppose there is an algorithm \omega which solves the OWMF(F) problem, such
that, the computational complexity of \omega is less than
        floor (floor(2^{(q-1)/3}/(2^{2/3}+2^{1/3}+1))/2).
By Lemma 1.1.6.
        |PossibleOnOne(\alpha_0)| \ge floor(2^{(q-1)/3} / (2^{2/3} + 2^{1/3} + 1)),
and, by definition
        | AllPossible | = | PossibleOnOne(\alpha_0) |
or
        | AllPossible | = floor(| PossibleOnOne(\alpha_0)| / 2).
Therefore, a priori, there is at least one element n in AllPossible such that, n is not
checked by the algorithm \omega and no operation is performed instead.
The algorithm \omega solves the OWMF(F) problem and as a result, for the unchecked n, it has
been decided if n is a solution to the problem or not.
Thus, by Lemma 1.1.1. (if n is a solution) or by Lemma 1.1.7. (if n is not a solution),
        (n^3, I_n - b_n) \neq 1,
and/or the divisibility of
        ((I_n - b_n) n^{\#} - 1) by n,
have been decided (by the algorithm \omega) without performing any operation.
By Lemma 1.1.9. and/or by Lemma 1.1.10. such a decision is impossible without
performing at least one operation.
As a result, the supposition that there is an algorithm \omega which solves the OWMF(F)
problem, such that, the computational complexity of \omega is less than
        floor (floor(2^{(q-1)/3}/(2^{2/3}+2^{1/3}+1))/2).
is false.
```

Four

Robustness

Jan Signs a Claimed Mathematical Breakthrough is Wrong

1. There is no proof.

CTRL + F adaptive

this paper's adaptive evaluation is actually **6x longer** than its non-adaptive evaluation ⁽²⁾

4 NON-ADAPTIVE EVALUATION

Having described our defense proposal, we begin by demonstrating that it has at least some potential utility: it effectively detects existing (unmodified) black-box query attacks. While there are many black-box (hard-label) attacks, they fall roughly into two categories:

- Gradient estimation attacks at their core operate like standard white-box gradient-based attacks (as described in Section 2.1). However, because they do not have access to the gradient, these types of attacks instead estimate the gradient by repeatedly querying the model.
- Boundary following attacks, in contrast, first identify the decision boundary of the neural network, at a potentially far-away point, and then take steps following the boundary to locate the nearest point on the boundary to the target image.

We evaluate against one representative attack from each category.

4.1 Attack Setup

For each attack studied, we use the *targeted* variant, where the adversary generates an adversarial example chosen so that the resulting adversarial example x is classified as a target class t and is within a distance ϵ of an original image x. The original image and target class are chosen randomly. We call an attack successful if the ℓ_∞ distortion is below $\varepsilon=0.05$. While most white-box work on CIFAR-10 considers the smaller distortion bound of $\epsilon=0.031\approx 8/255$, we choose this slightly larger distortion because black-box attacks are known to be more difficult to generate and so we give the adversary slightly more power to compensate.

NES [22] is one of the two most prominent gradient-estimation attacks (along with SPSA [39]). It estimates the gradient at a point by averaging the confidence scores of randomly sampled nearby points, and then uses projected gradient descent [30] to perturb an image of the target class until it is sufficiently close to the original image. In the hard label case, the confidence score for a point is approximated by taking a Monte Carlo sample of nearby points, and then computing the score for a class as the fraction of nearby points with that class.

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that the theorem
isn't completely
wrong..."

"actual proof!"



Proof. The proof will be released upon paper acceptance.





reproducible evaluation



pretrained models

Four Robustness
Signs a Claimed Mathematical Breakthrough is Wrong
2. There are many proofs.

We first present multiple adaptive attacks

various strong adaptive attacks

a variety of strong adaptive attacks,

A strong evaluation should be about *quality*, not *quantity*

evaluate 7 potential adaptive attacks.

Ten Signs a Claimed Mathematical Breakthrough is Wrong

3. The approach seems to yield something much stronger and maybe even false.



If the *proof still works* for a *theorem that is false,* there is clearly something wrong!



3. The approach seems to yield something much stronger and maybe even false.



If the evaluation still passes (all attacks fail) for a defense that is broken, there is clearly something wrong!

Building a minimally-altered, broken defense: the binarization test.



Building a minimally-altered, broken defense: the binarization test.



"Increasing Confidence in Adversarial Robustness Evaluations", Zimmermann et al., <u>https://arxiv.org/abs/2206.13991</u> 20

Building a minimally-altered, broken defense: the binarization test.



If the *evaluation were strong* it would break the non-robust defense

"Increasing Confidence in Adversarial Robustness Evaluations", Zimmermann et al., https://arxiv.org/abs/2206.13991 21

The binarization test identifies flawed evaluations.



"Increasing Confidence in Adversarial Robustness Evaluations", Zimmermann et al., https://arxiv.org/abs/2206.13991 22



Strong adaptive evaluations (which broke the defenses) pass the test.



Some adaptive attacks break defenses but remain quite weak.



"Increasing Confidence in Adversarial Robustness Evaluations", Zimmermann et al., https://arxiv.org/abs/2206.13991 25

Our test can have false positives.

original evaluations are not "completely wrong"







A convincing evaluation should *distinguish* robust defenses from broken ones!

Ten Signs a Claimed Mathematical Breakthrough is Wrong

4. The approach conflicts with a known impossibility result.

Theorem: Technique X won't help you solve P vs NP

RELATIVIZATIONS OF THE $\mathcal{P} = ? \mathcal{NP}$ **QUESTION***

THEODORE BAKER[†], JOHN GILL[‡] and ROBERT SOLOVAY[¶]

Natural Proofs

Alexander A. Razborov*

School of Mathematics, Institute for Advanced Study, Princeton, New Jersey 08540; and Steklov Mathematical Institute, Vavilova 42, 117966, GSP-1, Moscow, Russia

and

Steven Rudich[†]

Algebrization: A New Barrier in Complexity Theory

Scott Aaronson* MIT $\begin{array}{c} {\rm Avi \ Wigderson^{\dagger}} \\ {\rm Institute \ for \ Advanced \ Study} \end{array}$

Can we show such an *impossibility result* for adversarial ML?

Theorem: Technique X won't help you build a robust model

One attempt: a barrier for *detecting* adversarial examples.

Theorem: Detecting attacks won't help you build a robust model



"Detecting Adversarial Examples Is (Nearly) As Hard As Classifying Them", ICML 2022, https://arxiv.org/abs/2107.11630 30

We show a reduction from robust detection to classification.



We show a partial reduction from robust detection to classification.



efficient

\succ robust at distance ε

 $\succ inefficient (at inference) \\ \succ robust at distance \frac{\varepsilon}{2}$

Strongly robust detectors imply a *breakthrough in robust classification.*



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Can we build much more robust classifiers in **World 2**? (we don't know...)

Strongly robust detectors imply a *breakthrough in robust classification.*



Can we build much more robust classifiers in **World 2**? (we don't know...)

But any sufficiently robust detector implies a positive answer!

Many detectors *implicitly* claim such a breakthrough!



Many detectors *implicitly* claim such a breakthrough!



Optimistic view: this is a *breakthrough* in (inefficient) robust classification!



Pessimistic (*realistic?*) view: These detectors are *not robust!*



Robust detection is as hard as classification.

 $\Psi_n(Q) = A_n H_n(Q) e^{-Q^{2/2}}$ $\langle f \rangle = \int_{VV} \psi^*(r,t) f \psi(r,t) dV = \langle \psi | f | \psi \rangle$ $h = \frac{h}{2\pi}$ E= 100 $t = |\psi|^2 = \psi^{\prime\prime}(r,t)\psi(r,t) \qquad \hat{\mathcal{E}} = i\hbar\frac{\partial}{\partial t} \qquad \psi_0(Q) = \frac{1}{2\pi}e^{-Q^2}$ Δp,Δ×≥ħ/2 $\Delta x = \langle x^2 \rangle - \langle x \rangle$ $\Psi(r,t) = \Psi(r) \Psi(t)$ $Q = X/L \langle A \rangle = \sum |a_n|^2 A_n$ $\psi(r) + \sqrt{(r)} \psi(r) = \mathcal{E}\psi(r) \quad \psi(t) = e^{-i\mathcal{E}t/t}$

Robust classification

-4x + 7 = 15

Robust detection

Robust detection is as hard as classification.

 $\nabla \Psi - \Psi \nabla \Psi^{*}$ $\Psi_n(Q) = A_n H_n(Q) e^{-Q^{2/2}}$ $\langle f \rangle = \int_{M} \psi^{\dagger}(r,t) f \psi(r,t) dV = \langle \psi | \hat{f} | \psi \rangle \quad h = \frac{h}{2\pi}$ $\hat{\xi} = i\hbar \frac{\partial}{\partial t} \qquad \psi_0(Q) = \frac{1}{9r} \frac{e}{Q} - \frac{Q^2}{Q^2}$;t)=|ψ|²=ψ^{*}(r,t)Ψ(r,t) AD. AX≥h/2 $\Delta x = \langle x^2 \rangle$ $\Psi(r,t) = \Psi(r) \Psi(t)$ $\nabla^{\mathcal{Z}}\psi(r) + \sqrt{(r)}\psi(r) = \mathcal{E}\psi(r) \quad \psi(t) = e$

Robust classification



Robust detection

Four Robustness Signs a Claimed Mathematical Breakthrough is Wrong

4. Breakthrough results using only "weak" techniques.



detectors



denoisers, preprocessors



randomness

provably weak!

empirically weak

Treat your ML defense like a theorem!

Defense evaluations that aren't convincing are like theorems without proofs...

FourRobustnessTen Signs a Claimed Mathematical Breakthrough is Wrong

- 1. There is no adaptive attack (or no code). (no proof)
- 2. There are many partial adaptive attacks. (many proofs)
- 3. The evaluation fails to break a non-robust defense. (proof idea still holds for false theorems)
- 4. Breakthrough results using only "weak" techniques. (proof idea is believed/known to fail)